Jonas Kremer WS 2017/2018

Probability theory

Exercise Sheet 10

Exercise 1 *(4 Points)*
Let $(X_n)_{n\in\mathbb{N}}$ be a sequence of independent random variables, each Cauchy distributed with density

$$f(x) = \frac{1}{\pi}\frac{1}{1+x^2}, \quad x \in \mathbb{R}.$$

(a) For which α does

$$\frac{X_1 + \cdots + X_n}{n^\alpha}$$

converges in distribution?

(b) In case we have convergence in distribution, compute the limit distribution as well.

Exercise 2 *(4 Points)*
Let $(X_n)_{n\in\mathbb{N}}$ be a sequence of random variables with distribution function

$$F_n(x) = \begin{cases} 0, & \text{if } x < 0, \\ x\left(1 - \frac{\sin(2n\pi x)}{2n\pi x}\right), & \text{if } 0 \leqslant x < 1, \\ 1, & \text{if } x \geqslant 1. \end{cases}$$

(a) Prove that X_n converges in distribution to a random variable which is $[0,1]$-uniformly distributed.

(b) Prove that the sequences of the relative density functions $(f_n)_{n\in\mathbb{N}}$ does not converge.

Exercise 3 *(4 Points)*
Let $(X_n)_{n\geqslant 2}$ be a sequence of independent random variables on $(\Omega, \mathcal{F}, \mathbb{P})$ with

$$\mathbb{P}(X_n = -n) = \mathbb{P}(X_n = +n) = \frac{1}{2n\log n}, \quad \mathbb{P}(X_n = 0) = 1 - \frac{1}{n\log n}.$$

(a) Check either $(X_n)_{n\geqslant 2}$ satisfies the weak law of large numbers or not.

(b) Check either $(X_n)_{n\geqslant 2}$ satisfies the strong law of large numbers or not.

Exercise 4 *(4 Points, Talk)*
Prepare a talk on the proof of the Lévy continuity theorem.